**ZSTEVE Specialist Mathematics Exam 1**

Reading time: 15 minutes

Writing time: 60 minutes

No notes or calculator of any type are allowed.

40 marks are available

**N.B. this exam aims to extend the knowledge and test the adaptability of students to new concepts, and thus does not purely target its questions within the VCE study design scope. However, all questions should be doable with Specialist level skills. For further investigation, notes have been included in some extension questions.**

Exam written by Stephen Zhang, VCE Student in 2015.

1. Find the resolute of the vector in the direction of

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[2 marks]

1. [**further differential equations]**
2. Show that can be solution to (*this type of differential equation is called a second order linear homogenous equation)*

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1. Hence, for the general differential equation find an expression which can be solved for values of *k*. Give your answer as

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1. Let these solutions be , in any order. Show that must also be a solution, for arbitrary constants . (*This solution is the* ***general solution*** *to this differential equation!)*

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[2+2+2 = 6 marks]

1. Find the complex linear factors of [*hint – evaluate for z=-1*], expressing each factor in the form where is the principal Argument.

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[4 marks]

1. [**trigonometric substitutions]**
   1. By using the substitution where is a standard trigonometric function [**not an inverse trig function]**, find the antiderivative of . By restricting the values of , justify your mathematical reasoning.

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[3 marks]

* 1. Hence evaluate

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[1 mark]

1. An object of mass 1 kg is moving with initial velocity at time *t=0*, and is subjected to a net force .
   1. Find the terminal velocity of the object, i.e. .

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[3 marks]

* 1. Find the displacement from its initial position (let this be O) at any time *t*.

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[2 marks]

1. For the function , evaluate the value of at and thus give the equation of the tangent and normal at that point.

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[4 marks]

1. Using calculus, find an expression for the volume of the truncated cone with minimal and maximal radii respectively and height . Give your answer in the form where are real expressions.

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[5 marks]

1. Express in terms of the vectors and .

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[2 marks]

1. Find the area of the triangle ΔOAB for O(0,0), A(2,1), and B(1,4) using a vector method.

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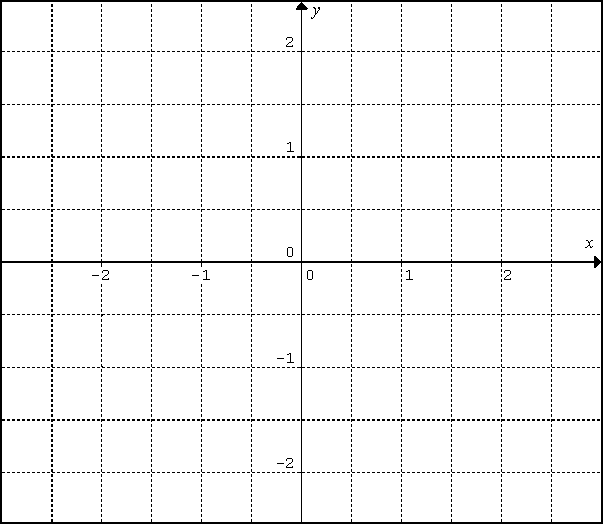
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[3 marks]

* 1. Sketch the direction field for and .



[2 marks]

* 1. Given that , and using the change of variable rule , find the general solution curve for this differential equation. (*These are called separable equations, and from what I hear are due to be introduced to the 2016 VCE Specialist study design)*

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[4 marks]

* 1. Using calculus if necessary, sketch the member of this family of curves which passes through on the diagram above.

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[2 marks]

**END OF EXAMINATION**

**Total marks: 40**